



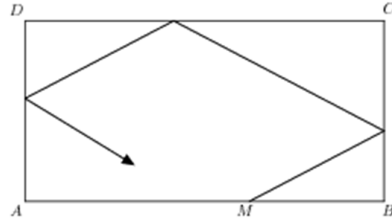
Starptautiskā konkursa
„Kengurs”
uzdevumi

22.03.2018.

11.-12. klases

25. A quadratic function $f(x) = x^2 + px + q$ is such that its graph intersects the x -axis and the y -axis in three different points. The circle through these three points intersects the graph of f in a fourth point. What are the coordinates of this fourth point?

- (A) $(0, -q)$ (B) (p, q) (C) $(-p, q)$ (D) $(-\frac{q}{p}, \frac{q^2}{p^2})$ (E) $(1, p + q + 1)$



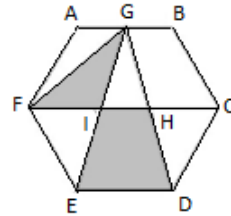
26. We are given a rectangular billiard table with sides of length $3m$ and $2m$. A ball is shot from the point M on one of the longer sides. It reflects once on every other side as shown. At what distance from point A will it hit the initial side if $BM = 1,2 m$ and $BN = 0,8 m$?

- (A) $1,2 m$ (B) $1,5 m$
(C) $2 m$ (D) $2,8 m$
(E) $1,8 m$

27. How many real solutions does the equation $||4^x - 3| - 2| = 1$ have?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

28. $ABCDEF$ is a regular hexagon. G is the midpoint of AB . H and I are the points of intersection of the segments GD and GE with FC respectively. What is the ratio between the area of the triangle GIF and the area of the trapezoid $IHDE$?



- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{4}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{\sqrt{3}}{4}$

29. There are 40 % more girls than boys in a class. The probability that a two-person delegation selected at random consists of a girl and a boy equals $\frac{1}{2}$. How many pupils are in this class?

- (A) 20 (B) 24 (C) 36
(D) 38 (E) This situation is not possible.

30. Archimedes calculated $15!$. The result is written on the board. Unfortunately two of the figures, the second and the tenth, are not visible. Which are these two figures?

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- (A) 2 and 0 (B) 4 and 8 (C) 7 and 4 (D) 9 and 2 (E) 3 and 8

Laiks uzdevumu risināšanai – 75 minūtes!

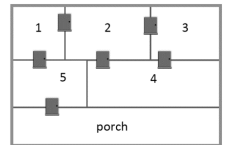
3 point problems

1. Which of the following numerical expressions has the highest value?
(A) $2 - 0 \cdot 1 + 8$ (B) $2 + 0 \cdot 1 \cdot 8$ (C) $2 \cdot 0 + 1 \cdot 8$
(D) $2 \cdot (0 + 1 + 8)$ (E) $2 \cdot 0 + 1 + 8$

2. We will call the number "modern" if the sum of its digits equals 2018. What is the first digit of the smallest of "modern" numbers?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 9

3. The figure shows the floor plan of Renate's house. Renate enters her house from the porch and walks through each door exactly once. In which room does she end up?



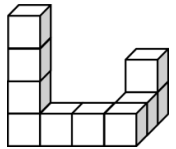
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4. Thor has seven stones and a hammer. Every time he hits a stone with the hammer it breaks into exactly five smaller stones. He does this several times. Which of the following numbers could be the number of stones he may end with?

- (A) 17 (B) 20 (C) 21 (D) 23 (E) 25

5. The shape shown is made of 10 cubes glued together. The shape is dipped into a bucket of paint covering the surface entirely. How many of the cubes will be painted on exactly four of their faces?

- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

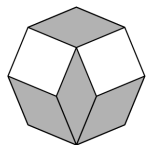


6. The following two statements are true: «Some aliens are green, the others are purple» and «Green aliens live only on Mars». Therefore, it logically follows that

- (A) all aliens live on Mars
(B) only green aliens live on Mars.
(C) some purple aliens live on Venus.
(D) all purple aliens live on Venus.
(E) no green aliens live on Venus.

7. Four identical rhombuses and two identical squares are put together to make a regular octagon. What is the measure of the larger angle of each rhombus?

- (A) 135° (B) 140° (C) 144° (D) 145° (E) 150°

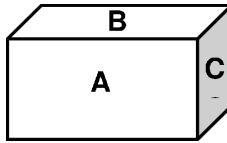


8. There are 65 balls in a box. 8 are white and the rest of the balls are black. In one move, at most 5 balls can be taken out of the box. It is not allowed to put any balls back in the box. What is the smallest number of moves needed to ensure that at least one white ball is taken out?

- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

9. The faces of a rectangular brick have areas A, B and C as shown. What is the volume of the brick?

- (A) ABC (B) \sqrt{ABC}
 (C) $\sqrt{AB + BC + CA}$ (D) $\sqrt[3]{ABC}$ (E) $2(A + B + C)$

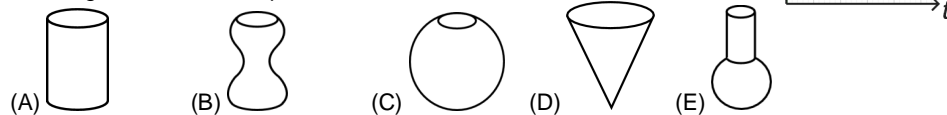


10. In how many ways can the number 1001 be written as the sum of two primes?
 (A) none (B) one (C) two (D) three (E) more than three

4 point problems

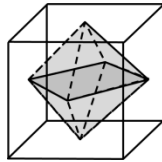
11. Two cubes of volumes V and W intersect. The part of the cube of volume V which is not common to the two cubes is 90% of its volume. The part of the cube of volume W which is not common to the two cubes is 85% of its volume. What is the relationship between V and W ?
 (A) $V = \frac{2}{3} W$ (B) $V = \frac{3}{2} W$ (C) $V = \frac{85}{90} W$ (D) $V = \frac{90}{85} W$ (E) $V = W$

12. A vase is filled up to the top with water, at a constant rate. The graph shows the height h of the water as a function of time t . Which of the following could be the shape of the vase?

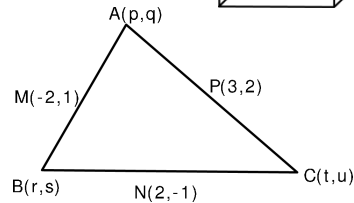


13. $|\sqrt{17} - 5| + |\sqrt{17} + 5| =$
 (A) 10 (B) $2\sqrt{17}$ (C) $\sqrt{34} - 10$ (D) $10 - \sqrt{34}$ (E) 0

14. An octahedron is inscribed in a cube of side length 1. The vertices of the octahedron are at the center of the faces of the cube. What is the volume of the octahedron?
 (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$ (E) $\frac{1}{8}$



15. The vertices of a triangle are $A(p, q), B(r, s)$ and $C(t, u)$ as shown. The midpoints of the sides of the triangle are the points $M(-2, 1), N(2, -1)$ and $P(3, 2)$. What is the value of $p + q + r + s + t + u$?
 (A) 2 (B) 5/2 (C) 3
 (D) 5 (E) none of these



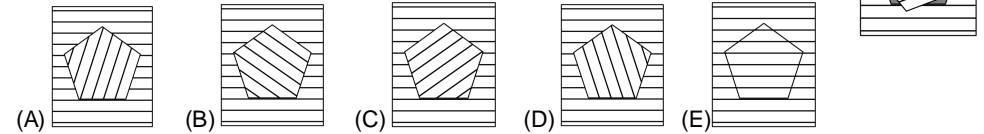
16. Five predictions were made before the football match between «Real Madrid» and «Manchester United»:

1. The game will not end in a draw;
2. «Real Madrid» will score;
3. «Real Madrid» will win;
4. «Real Madrid» will not lose;
5. Three goals will be scored.

What was the final score of the match «Real Madrid» - «Manchester United» if exactly three of the predictions came true?

- (A) 3-0 (B) 2-1 (C) 0-3
 (D) 1-2 (E) this situation is not possible

17. We cut out a regular pentagon from a lined piece of paper. In each step we rotate the pentagon counter clockwise around its centre by 21° . The situation after the first step is shown. What will we see when the pentagon first fits back in the hole?

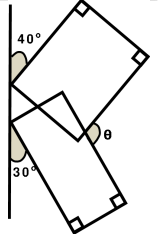


18. Which of these five numbers does not divide $18^{2017} + 18^{2018}$?
 (A) 8 (B) 18 (C) 28 (D) 38 (E) 48

19. Three of the five cards shown are given to Nadia and the rest to Riny. Nadia multiplies the three values of her cards and Riny multiplies the two values of his cards. It turns out that the sum of the two resulting products is prime. What is the sum of the values of Nadia's cards?
 (A) 12 (B) 13 (C) 15 (D) 17 (E) 18

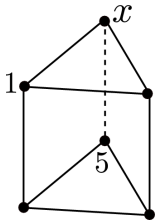


20. Two rectangles are inclined to the vertical line at angles 40° and 30° as shown. What is the measure of the angle θ ?
 (A) 105° (B) 120° (C) 130°
 (D) 135° (E) None of these



5 point problems

21. The prism in the picture is formed of two triangles and three squares. The six vertices are numbered from 1 to 6 in such a way that the sum of the four vertices of each square is the same for all three squares. Numbers 1 and 5 are already shown. What number is at the vertex labeled x ?
 (A) 2 (B) 3 (C) 4
 (D) 6 (E) the situation is impossible



22. m and n are the roots of the equation $x^2 - x - 2018 = 0$. What is the value of $m + n^2$?
 (A) 2016 (B) 2017 (C) 2018 (D) 2019 (E) 2020

23. Four brothers named A, B, C and D have different heights. They state the following:
 A: «I am neither the tallest nor the shortest».
 B: «I am not the shortest».
 C: «I am the tallest».
 D: «I am the shortest».

Exactly one of them is lying. Who is the tallest?

- (A) A (B) B (C) C
 (D) D (E) We do not have enough information

24. Let f be a function such that $f(x + y) = f(x)f(y)$ for all integers x and y . If $f(1) = 1/2$, find the value of $f(0) + f(1) + f(2) + f(3)$.
 (A) 1/8 (B) 3/2 (C) 5/2 (D) 15/8 (E) 6